STUDY OF CRACK PROPAGATION IN THE SPECIMEN RECOMMENDED BY RILEM TC 162 BASED ON LINEAR ELASTIC FRACTURE MECHANICS

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1. Introduction

The toughness of fiber reinforced concrete is normally determined using unnotched beams subjected to third-point loading. The problems encountered in such tests and in the analysis of their results have led to the recent proposal of a notched beam under center-point load by RILEM TC 162. Nevertheless, such specimens have been used for many years in the determination of fracture mechanics parameters. Along these lines, a linear elastic fracture mechanics (LEFM) based analysis is performed on the specific geometry recommended by RILEM TC 162 in the present work with the objective of providing the equations needed and of reaching preliminary conclusions regarding tests with this geometry.

The beam used in the RILEM TC 162 recommendation has the dimensions of length * 550 mm, span (*S*) = 500 mm, width (*B*) = 150 mm and depth (*W*) = 150 mm. A notch is cut at mid-span with a length (a_0) of 25 mm. The specimen is loaded in the three-point bending or center-point loading configuration, as shown in Fig. 1. The crack mouth opening displacement (CMOD) is determined by placing transducers across the mouth of the notch, normally at a small distance (*d*) from the bottom of the beam.

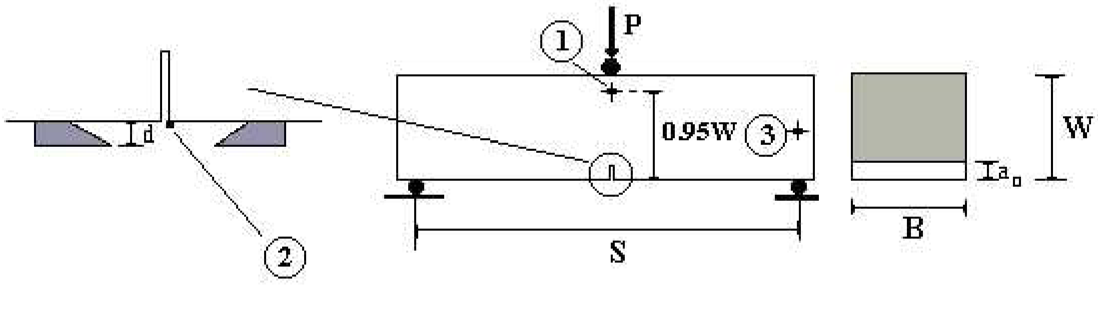

Figure 1. Specimen geometry and the reference points for determining displacements

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2. Methodology

Within the framework of linear elastic fracture mechanics (LEFM), the loads and displacements associated with crack propagation in notched specimens can be studied quite easily (e.g., Anderson, 1991; Karihaloo, 1995; Bazant and Planas, 1997), especially in the case of Mode I (tensile) cracking in simple geometries, such as the present notched beam specimen (Fig. 1). For many such geometries, the relevant equations are given in fracture handbooks but the specimen geometry studied here is not one of them. Therefore, elastic fracture analysis has to be performed using an adequate technique in order to obtain the geometry-dependent equations relating the stress intensity factor and the displacements to the applied load as a function of crack length. Here, finite element analysis is used to determine the equations for the stress intensity factor, vertical displacements and crack mouth openings.

The numerical computations of the LEFM equations were performed through two-dimensional plane stress analysis, using the FRANC2D/L program (James and Swenson) developed at Cornell University and Kansas State University. A beam length of 550 mm was considered and a concentrated load was applied at mid-span. The beam was modeled with a mesh composed of quadrilateral and triangular quadratic isoparametric finite elements, with about 37000 degrees of freedom. The notch and the propagating crack were represented as mathematical cracks of zero thickness, and the crack-tip singularity was handled through a rosette of eight triangular quarter-point elements with a radius of 0.03 mm around the "current" crack tip.

The crack is propagated along the notch plane and the stress intensity factor (K_I) is calculated for several crack lengths. At each step, several displacements have been determined:

- the vertical displacement of a point that is located at a distance of $0.05W$ below the load-point (denoted as 1 in Fig. 1), with reference to the mid-height above the supports (denoted as 3 in Fig. 1). This measurement (δ_1) will correspond to readings obtained between a reference point that is slightly below (5% of the beam depth) the loading-point and a rigid yoke supported at mid-height over the supports.
- the vertical displacement of the edge of the notch mouth (denoted as 2 in the inset of Fig. 1), with reference to the mid-height above the supports (denoted as 3 in Fig. 1). This measurement (δ_2) will correspond to the readings obtained between the notch edge at the bottom face of the beam and a rigid yoke supported at mid-height over the supports.
- the crack opening displacement (*CMOD*) given by the horizontal separation between the two bottom edges of the crack.
- the nominal crack mouth openings ($CMOD^m$) at different distances (d) from the bottom face, determined as the increase in the horizontal separation between the knife edges shown in the inset of Fig. 1. Each knife edge was modeled with a secondary mesh of small rigid elements conveniently attached to the bottom of the beam. The distance between the knife edges was taken to be 40 mm. The $CMOD^m$-values correspond to crack mouth openings measured by sensors that are placed on knife-edges or supports of thickness d.

Denoting the total crack length (i.e., notch length + crack extension) as a, the relative non-dimensional crack length, which will be used in most of the following discussions, is defined as $\alpha = a/W$, where W is the depth of the beam (in the present case, 150 mm). The numerical computations were performed over a relative crack length range of $0.167 \le \alpha \le 0.833$ (corresponding to a total crack length range of 25 mm $\le a \le$ 125 mm).

3. Results

The following results have been verified numerically for the range of $0.167 \le \alpha \le 0.833$ but should also be valid for values of α that are slightly outside this range.

3.1. Stress Intensity Factors

The stress intensity factor (K_I) for beam geometries undergoing Mode I fracture is often given in the form of Eq.(1):

$$K_I = \frac{1.5PS\sqrt{\pi.a}}{BW^2} f(\alpha) \tag{1}$$

where P is the applied load, a is the total crack length, α is the relative crack length, and S, B and W are the span, depth and thickness of the beam. The dimensionless geometry-dependent function, $f(\alpha)$, has been obtained by fitting the results of the computations with a reciprocal quadratic function:

$$f(\alpha) = \frac{1}{1.377 - 1.243\alpha - 0.247\alpha^2} \tag{2}$$

For a situation where LEFM is applicable, considering that the maximum value of K_I is the fracture toughness or critical stress intensity factor K_{Ic}, we can obtain the maximum load for each crack length by using the fracture criterion $K_I = K_{Ic}$ in Eq. (1).

3.2. Vertical Displacements

The load-point displacement can be calculated from the equation for K_I, as explained, for example, by Bazant and Planas (1997). However, in the present work, the displacements are determined directly from the finite element analysis.

The displacement of point 1 (Fig. 1), located at $0.05W$ below the loading-point, relative to the supports is given by the following equation, which has been obtained by fitting the numerical results:

$$\delta_1 = \frac{1.5PS^2}{EBW^2} V_1(\alpha) \tag{3}$$

where:

$$V_1(\alpha) = \frac{1}{1.823 - 3.697\alpha + 1.876\alpha^2} \tag{4}$$

Similarly, the displacement of point 2 (inset of Fig. 1), at the edge of the notch mouth, relative to the supports is given by the following equation, which has the same form as Eq. (3):

$$\delta_2 = \frac{1.5PS^2}{EBW^2} V_2(\alpha) \tag{5}$$

where:

$$V_2(\alpha) = \frac{1}{1.776 - 3.623\alpha + 1.854\alpha^2} \tag{6}$$

In Fig. 2, a comparison of the displacement functions $V_1(\alpha)$ and $V_2(\alpha)$ is presented. It can be seen that the values are practically the same, especially up to the value of α=0.5. This indicates that for practical purposes, both methods of measurement lead to the same results.

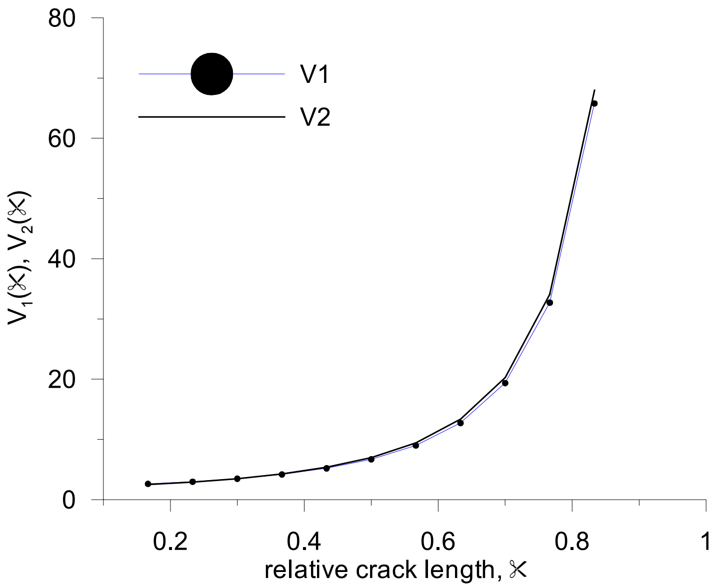

Figure 2. Comparison of central deflections corresponding to different heights

3.3. Crack Mouth Opening Displacements

As in the case of the load-point displacement, the *CMOD* can also be calculated from the equation for K_I, as explained by Bazant and Planas (1997). However, in the present work, the *CMOD* values are also determined directly from the finite element analysis, which is also used to compute the nominal $CMOD^m$-values corresponding to different knife-edge thicknesses *d*.

The *CMOD* is given by the following equation, which has been obtained by fitting the numerical results:

$$CMOD = \frac{6PSa}{EBW^2} U(\alpha) \qquad (7)$$

where:

$$U(\alpha) = \frac{1.021 - 0.760\alpha}{1 - 2.149\alpha + 1.162\alpha^2} \qquad (8)$$

As explained earlier, the crack opening displacement is normally measured in laboratory tests at a small distance below the crack mouth. The effect of this distance *d* (see inset of Fig. 1) is analyzed by considering the relation between the "real" *CMOD* and the nominal $CMOD^m$-values obtained experimentally as:

$$CMOD = c_c CMOD^m \qquad (9)$$

where c_c is the theoretical correction that has to be applied for converting the nominal value to the *CMOD*.

The *CMOD* correction factor, c_c, has been obtained through numerical analysis for the knife-edge thickness range of 0 # *d* #10 mm, for a beam depth W = 150 mm. The correction varies with the crack length, as seen in Fig. 3; the value decreases asymptotically as the crack length increases.

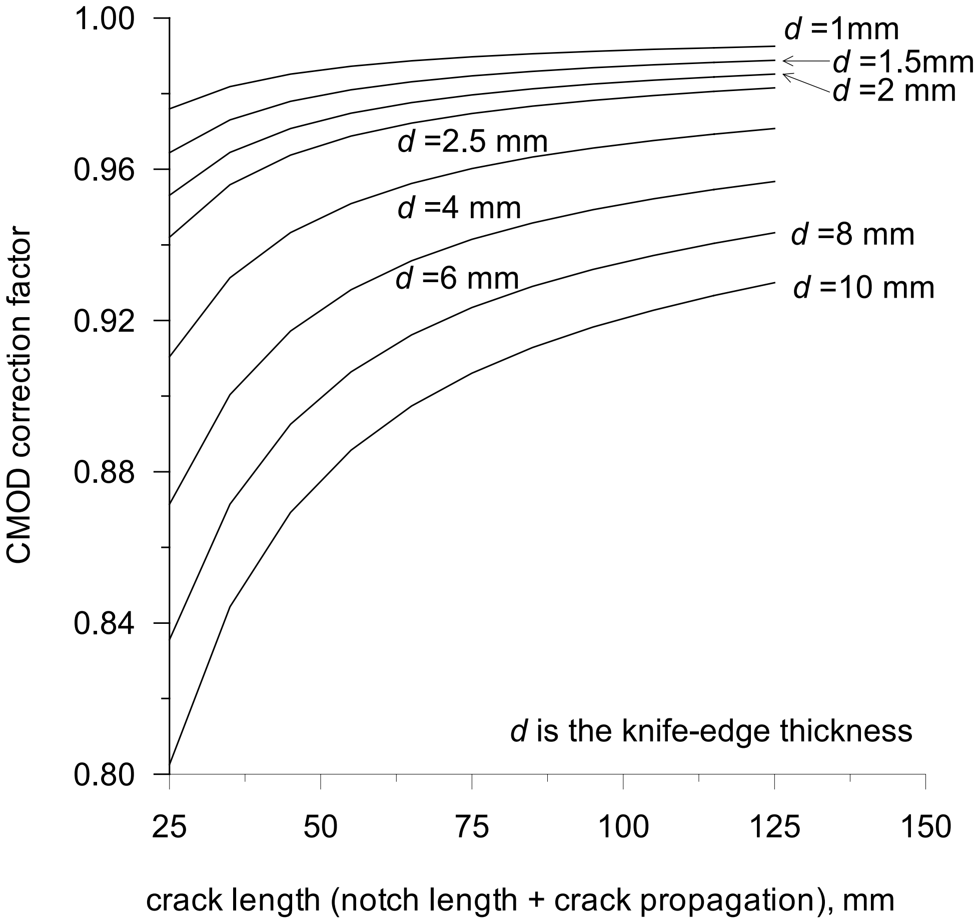

Figure 3. Correction factors for different knife-edge thickness and crack lengths

It can be seen that for *d* # 2 mm, the error in the measurement is less than 5%, which can be acceptable in most tests. For *d* = 10 mm, the error varies from values of 20% to 8%, which are quite significant. The trends in Fig. 3 have been fitted with power laws of the form:

$$c_c = c_1 c_2^{1/\alpha} \qquad (10)$$

where c_1 and c_2 are constants that depend on the value of *d*, and are given in Table 1.

d (mm)	c_1	c_2
1.0	0.996681	0.996513
1.5	0.994989	0.994815
2.0	0.993275	0.993145
2.5	0.991544	0.991502
4.0	0.986252	0.986729
6.0	0.978993	0.980707
8.0	0.971555	0.975037
10.0	0.963978	0.969687

Table 1. Values of the constants in Eq. (10)

4. Implications for Tests on Fiber Reinforced Concrete Beams

4.1. Applicability of LEFM to Fiber Reinforced Concrete

The applicability of LEFM to fiber reinforced concretes, and to concrete, in general, is limited (Bazant and Planas, 1997). Two important hypotheses of LEFM are not satisfied: the existence of singular stresses at the crack-tip and the existence of a zone of negligible size where all the energy is dissipated during fracture. Nevertheless, it provides a good approximation to the behavior of large elements and can give an idea of the response of laboratory-scale specimens.

For fiber concretes, the fracture process zone or bridging zone that would occur in front of the propagating is of considerable size, especially in concretes with normal-size fibers (i.e., not microfibers). Therefore, the equivalent LEFM crack would have its tip within this zone and have a length smaller than the real crack length.

A much better use of the fracture mechanics to fiber concretes would be through a cohesive crack model, using the stress-separation curve of the material under tension. R-curve modeling based on the LEFM equations also seems to be promising but with much less scope.

4.2. Equations for Deflection and *CMOD*

As mentioned earlier, the presence of the fracture process zone eliminates the possibility of directly applying LEFM to fiber concretes. Nevertheless, the equivalent crack concept can be utilized along with the LEFM equation presented in section 3. The equivalent crack length corresponding to each load should, however, be determined through an appropriate approach, which is not always easy.

The *CMOD* correction factor discussed in sub-section 3.3 can be used to transform experimentally-determined crack openings to the *CMOD*. However, the exact correction cannot be determined since the equivalent crack length is needed. Therefore, corrections for different equivalent crack lengths can be considered, as in Table 2, in order to propose a correction factor for each knife-edge thickness *d* and limitations on the knife-edge thickness itself.

knife-edge thickness, d (mm)	Correction factor c_c for different LEFM crack lengths, a			
	25 mm (negligible crack propagation, pre-peak regime)	50 mm	100 mm	125 mm (ligament length of 25 mm, end of test)
1.0	0.976	0.986	0.991	0.993
1.5	0.964	0.979	0.987	0.989
2.0	0.953	0.973	0.983	0.985
2.5	0.942	0.966	0.979	0.982
4.0	0.910	0.947	0.967	0.971
6.0	0.871	0.923	0.951	0.957
8.0	0.836	0.900	0.935	0.943
10.0	0.803	0.877	0.921	0.930

Table 2. Correction factors for different knife-edge thicknesses and LEFM crack lengths

Considering a maximum error of 5% (i.e., a correction factor of 0.95), it can be seen from Table 2 that the knife-edge should not have a thickness of more than 2.5 mm. Also, within this thickness range, the correction factor can be taken as one constant in the pre-peak regime and another constant in the post-peak regime (e.g., the value corresponding to a = 100 mm). Accordingly, for d = 2 mm, we would a pre-peak correction factor of 0.953 and a post-peak correction factor of 0.983.

For thicknesses larger than 2.5 mm, the correction factor has to be interpolated based on some guidelines for crack lengths. For example, for d = 5 mm, the pre-peak correction factor would be about 0.891 (i.e., error of about 11%) and in the post-peak regime, the correction factor would gradually decrease to about 0.959 (i.e., error of about 4%).

5. References

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