Abstract
Crack propagation in residual stress-strain fields is usually observed around holes in riveted structures, like fuselages for example. The lug-bolt assembly is a well known example in which this kind of propagation is commonly found. A simulation of crack evolution in a lug-bolt contact region is presented in this paper. The analysis is done through the use of interface elements and the dynamic relaxation (DR) solver. In this type of problem, a residual stress state is introduced via a complete load-unload cycle. An incremental load scheme drives the elasto-plastic analysis. The residual fields are then considered as a load case. In addition, an eccentric load is applied to the bolt and a crack is started at the stress concentration zone. Due to crack propagation, the compliance of the specimen changes, which in turn changes the contact loading direction. Insights into the influence of the residual stress field and the contact friction conditions on crack propagation are also presented.

1. Introduction
In a lug-bolt assembly (Figure 1) the hole is first cold-worked to create a compressive state of residual stress along its circumference. This process is carried out to improve the crack initiation and crack extension resistance when external loads are transferred through the bolt. In addition, when eccentric overloads are applied to the bolt, a crack might start at a stress concentration zone. Due to crack propagation, the compliance of the specimen changes, and consequently there is a change in the contact loading direction. The influence of the residual stress field and the contact friction conditions on crack propagation are also investigated. To simulate the behavior of the lug-bolt system some approximations and assumptions are required. There are two well defined phases in this analysis. The first is related to the cold-working process of the hole, the second to the crack modeling in a residual stress field. The basic hypothesis being considered here is that, in the first phase, an elasto-plastic analysis is used to generate the residual stress-strain fields. These fields can be then mapped onto the second phase of the analysis. In the second phase, linear elastic material behavior is assumed. The effects of the residual fields are then treated as equivalent body forces. Within the assumption of linear elasticity,
superposition is valid and external loads can be added to the equivalent residual body forces. The propagation of a crack starting in the region of high tensile stress is then studied. Linear Elastic Fracture Mechanics is used to guide the crack extension, and remeshing capabilities are employed to model the geometry evolution. However, the contact region between the lug and the bolt is constantly changing due to the loss of stiffness as the crack evolves. Therefore, although the problem can be seen as a linear one from the material and crack propagation point of views, it is actually nonlinear in terms of the load transfer mechanism. To model the contact accordingly, interface elements (or gap elements) are placed along the lug-bolt interface and the dynamic relaxation solver is employed to obtain an equilibrated solution for each stage of the crack trajectory.

![Figure 1 - Lug-bolt problem.](image)

### 2. The Lug-bolt Problem Simulation

#### 2.1. Modeling Strategy

The first phase of the analysis is characterized by the introduction of a residual compressive stress field around the hole. To reproduce the “cold-working” process of the hole, an elastic material property is assigned to the internal bolt while an elastoplastic material property is defined for the lug. A fictitious thermal load is assigned to the elements comprising the bolt. This introduces an increase in the diameter of the hole. Consequently, yielding of the lug material may occur. Residual stresses and strains can then be computed after a complete load-unload cycle.

In the second phase of the analysis, an eccentric external load case is superimposed on the initial stress/strain fields. Basically, two approaches are possible for the external loading: (1) apply load directly to the lug (e.g. distributed load); and (2) apply load to the bolt and solve the contact problem. In the first approach, the load would be an educated guess of the contact force distribution. The bolt, in this case, is not described in the mesh model. A direct linear elastic analysis can be used to obtain the solution. In (2), the mesh should contain a description of the lug, the bolt and the interface between them. Interface elements (or gap elements) must be defined. A linear analysis is no longer appropriate and dynamic relaxation is used to solve the contact/friction problem. It should be observed that this approach is much more accurate than (1), and therefore, is employed in the numerical example to follow.
After the first cycle of analysis under combined eccentric and residual load, a crack can be initiated from the lug-bolt interface. The analysis procedure is repeated. Within the hypothesis of linear behavior, stress-intensity factors can be computed and used to predict a new crack configuration. The crack increment is specified by the analyst. The displacement correlation technique is employed in this case to compute stress intensity factors [Bittencourt 1992]. Some additional features are also necessary to model the lug-bolt problem as described before: elasto-plastic analysis, a residual stress-strain mapping mechanism, and nucleation of a crack starting from an interface.

2.2. Elasto-plastic Analysis
Since the elasto-plastic analysis is nonlinear, an iterative scheme is used. The full Newton-Raphson method combined with load steps has been employed for this purpose. To solve the equations of equilibrium at each step, the user has the choices of the direct method with triangular decomposition, or the dynamic relaxation method. It should be pointed out that dynamic relaxation is only being employed as a solver in the Newton-Raphson approach. The direct method is much more efficient and should be used in this context.

Linear isotropic or kinematic hardening rules and elastic perfectly plastic behavior are allowed in the elasto-plastic analysis. Presently, the analysis is limited to associated isotropic plasticity material models. The von Mises criterion is the only yield criterion considered.

Complete unloading of the structure is considered to generate residual stress and strain fields. These fields can be stored in an ASCII file (filename.RSI). The residual field file is generated for the elasto-plastic materials only. The influence of a material type on the unloading process may be neglected, which is an essential feature for the problem here described. The material of the lug is subjected to unload while the material of the bolt is not.

There are no restart capabilities currently available. More details of the theoretical background and implementation can be found in [Bittencourt 1993].

2.3. Residual Stress-Strain Mapping Mechanism
The elasto-plastic capability is used to generate the residual stress and strain fields. A distribution of points with respective values of the stress and strain components is stored. These values then can be then mapped onto a "new problem". The stored information is read at the Gauss point locations of the new model and equivalent residual (body) forces are generated. Currently, the residual fields are considered as an initial stress/strain state. A more detailed description of the residual loading process is found in [Bittencourt 1993].

2.4. Nucleation of a Crack From Interfaces
To model a crack starting from an interface between two materials representation issues need to be addressed first. The material boundary should be preserved. This fact adds some complications to the correct representation of the model (or mesh), especially when interface elements are used along a material boundary.

This representation problem has been addressed and an edge crack can now be nucleated from the boundary between two materials when interface elements are present. A new procedure to allow such a representation has been implemented in the program FRANC2D. This procedure makes use of the Winged-Edge data structure, as well as Euler operators which update the topology. It should be pointed
out that FRANC2D was capable of handling such material boundary problems in the past, but the possibility of interface elements along the material boundary was not allowed. In the new procedure, the internal material boundary with interface elements is treated as a regular external boundary. However, the philosophy employed in FRANC2D to nucleate a crack is preserved. Elements lying in the crack path are deleted, then the topology modifications to represent a crack are introduced, and finally remeshing takes place [Wawrzynek 1987, Wawrzynek 1991]. The procedure addressed here is related to the topology modifications to accommodate the crack and preserve material boundaries along with interface elements. The other elements of the crack nucleation strategy are kept intact.

Figure 2 - Nucleation of cracks from a line of interface elements.

The procedure in question comprises a sequence of steps (Figure 2). First, the crack starting point along the interface (a corner node set) is indicated. Then the position of the crack tip is specified. The program detects automatically which side of the interface needs to be modified to accommodate the new crack. The node (vertex) that is on the crack side is selected. It should be mentioned that the interfaces considered may have zero thickness, and therefore, no distinction between the two vertices of a node set can be made through the coordinates of these nodes. The vertex lying on the side of the new tip is automatically selected. For clarity, the selected vertex is named *Work Vertex*, while the opposite vertex is called *Base Vertex*. For the *Work Vertex*, a “split vertex make edge” operation is performed. A new vertex (*Vertex 1*) and a new edge (*Edge 1*) are created. A new face (*Face 1*) can now be created with the introduction of another edge (*Edge 2*) linking *Vertex 1* to the *Base Vertex* (“make face edge” operation). *Face 1* is set to be a free face, or in other
words, a hole. The Crack tip Vertex is introduced by performing a “split edge make vertex” operation on Edge 1. A new edge (Edge 3) is introduced with the Crack tip Vertex. Edge 1 and Edge 3 are then split by a sequence of “split edge make vertex” operations which defines both crack sides. Finally, the Crack tip Vertex is moved to the position previously specified. It should be noticed that the Work Vertex and Vertex 1 define the mouth of the new crack. So the representation of a cohesive crack, as described in a later chapter, can be easily introduced by placing interface elements along the crack line between the Crack tip Vertex and the vertices defining the mouth of the crack.

2.5. Input Data and Results

The geometry and boundary conditions of the lug-bolt example are shown in Figure 3a. Five types of material are defined:

![Figure 3](attachment:image.png)

(a) Initial geometry and boundary conditions (units in).
(b) Initial finite element mesh.

**Material 1** - Elastic isotropic:
\[ E = 70,000 \text{ ksi} \quad \nu = 0.25 \quad \alpha = 10^{-4} / \text{°C} \quad \Delta T = 100 \text{ °C} \]
(Bolt material in phase 1 of simulation)

**Material 2** - Elasto-plastic isotropic:
\[ E = 7,000 \text{ ksi} \quad \nu = 0.25 \quad E_t = 224 \text{ ksi} \quad \sigma_Y = 24.3 \text{ ksi} \]
isotropic hardening Mises criterion
(Lug material in phase 1 of simulation)

**Material 3** - Elastic isotropic:
\[ E = 7,000 \text{ ksi}, \quad \nu = 0.25 \]
Paris Parameters: \[ C = 10^{-8} / \text{µin} \quad m = 2.1 \]
(Lug material in phase 2 of simulation)

**Material 4** - Elastic isotropic:
\[ E = 10,000 \text{ ksi}, \quad \nu = 0.25 \]
(Bolt material in phase 2 of simulation)

**Material 5** - Gap element (µ = 0.0 or µ = 0.1), µ is the coefficient of friction.
The goal in the first phase of the analysis is to generate the residual stress and strain fields. The cold-working of the hole is artificially achieved by specifying a change in the temperature of the bolt. A uniform temperature variation of 100 °C and a coefficient of thermal expansion (\( \alpha \)) of \( 10^{-4} / \degree \)C are used to define the thermal element load. This load provides a change of 1% in the diameter of the hole. Then, a nonlinear elasto-plastic finite element analysis is performed. The default values of, (1) number of load increments (set to 3), and (2) equilibrium tolerance (set to \( 10^{-4} \)) are employed in this simulation. The finite element mesh is displayed in Figure 3b. The residual fields are stored in a file for later use. A contour plot of the minimum principal residual stresses are provided in Figure 4a. Figure 4b shows a line plot of the stress component \( \sigma_{xx} \) for the considered residual field. The simulation of the cold-working process of the hole in the lug is now complete.

In the second phase of the analysis, the previously generated residual stress and strain fields are considered as initial stress and strain fields respectively. Linear elasticity is now assumed. The lug material is set to material 3, while the bolt is set to material 4. In addition, gap elements are necessary to model the contact between the bolt and the lug (material 5). The next step is the specification of loads for the second phase of the analysis. Two load cases are then specified. Load case 1 incorporates the residual fields. Load case 2 specifies an eccentric point load at the center of the bolt (\( F_x = -100.0 \) kips, \( F_y = 100.0 \) kips). The dynamic relaxation method is employed to compute the results (tol = 0.0005, max iterations = 3000). The deformed configuration obtained for load case 2 is depicted in Figure 5.

A crack is then started from the interface between the bolt and the lug in the radial direction. The initial crack length is set to 0.24 inches. Remeshing capabilities are used to regenerate the finite element mesh for the new geometry. Dynamic relaxation analyses are performed (using same parameters as before) for each new crack configuration. Three distinct cases of residual load and contact condition combinations are considered. First, frictionless contact (\( \mu = 0.0 \)) and no residual loads are assumed. The residual stress and strain fields obtained in phase 1 (1% \( \Delta \) diameter: change of 1% in the hole diameter) are introduced. The hypothesis of frictionless contact is kept though. The crack propagation for this case is shown in Figure 6. Finally, contact friction (\( \mu = 0.1 \)) is introduced. The crack trajectories are similar for the three cases. However, stress intensity factor values are substantially different. Displacement correlation values of stress intensity factors are plotted with respect to crack length (Figure 7) for the three cases. Assuming that the Paris fatigue model (\( da/dN = C (\Delta K_{eff})^m \)) is valid, the life of the specimen with respect to crack length (Figure 8) can be obtained by simply integrating the \( K_I \) curves in Figure 7 (for sake of simplicity \( K_I \) is used instead of \( K_{eff} \)):

\[
N = \int_{a_{i-1}}^{a_i} \frac{1}{C(\Delta K_{eff})^m} da
\]

It is clear that the cold-forming process enhances the life of the specimen. This is basically due to a reduction of \( K_I \) values in the early stages of crack propagation. The non-consideration of friction for this problem is conservative in terms of the remaining life, as can be inferred from the fatigue life plots in Figure 8. The introduction of friction allows the structure to better redistribute the applied eccentric load, which in consequence reduces the crack driving force. Alternatively, one may argue that the loss of energy due to friction reduces the energy available for driving the crack.
Figure 4 (a) - Color contour of minimum principal residual stress field for a diameter expansion of 1%.
(b) - Line plot (A-B) of residual stress component $\sigma_{xx}$ for a diameter expansion of 1%.

The conclusions above cannot be taken as general ones in terms of the friction influence and crack trajectories. A different initial crack location with respect to
the residual field and/or to the friction zone might provide a different result. However, instead of invalidating the analysis presented here, the last argument just emphasizes the need for a general capability like the one described, where different hypotheses can be easily checked and their effects on life predictions evaluated.

(Fx = -100, Fy = 100 kips)

Figure 5 - Deformed shape for eccentric load (Magnification factor = 8.87).

2.6. Limitations of the Current Lug-Bolt Analysis Capability

- The presented analysis strategy is currently limited to two materials. In the case of the example presented, a single bolt, or hole is allowed.
- The initial assumption of linear behavior in the second phase of the problem needs to be considered with appropriate judgment because residual stress redistribution is not considered during crack nucleation and propagation.
- Only one crack starting from a line of interface elements is possible.
- Interface elements are not considered in the elasto-plastic analysis.
- In the process of storing the residual stress-strain fields, it is currently assumed that only one elasto-plastic material is modeled.
- The residual stress-strain field transfer mechanism usually requires a highly refined mesh description for the elasto-plastic phase of the solution.

3. Conclusions

In this paper, an example problem (lug-bolt assembly) has been analyzed through the use of DR together with interface elements. This problem is very complex and some simplifying assumptions are necessary. However, the goal here has been to show how different tools can be assembled so that the problem can be addressed in a consistent and usable manner on a graphics workstation. The use of different tools incorporated within FRANC2D, such as the residual field mapping mechanism and the elasto-plastic analysis capability, has been crucial for the modeling presented here. Together with interface elements and DR, they have provided a complete procedure to address the proposed problem. Improvements, like for example the consideration of redistribution of the residual fields during crack propagation, might be necessary to compute more accurate fatigue-life estimates, however.
Figure 6 - Crack propagation for 1% -diam., residual load, and no friction (Magnification factor = 10).

Figure 7 - History of Mode I Stress Intensity Factors.
Figure 8 - Fatigue life predictions.

References


