Fracture Modeling of Concrete Using Two Different Microstructural Mechanics Approaches

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Abstract

Here we present two microstructural mechanics approaches to model the development and propagation of fracture in concrete. One of the two approaches is the Delft Lattice Model [14] where the microstructure of concrete can be projected on a lattice and corresponding properties are assigned to relevant elements in the lattice. Another approach is the Microstructural Micropolar Model [7]. In this model, concrete is assumed to have an underlying microstructure of lattice type and the stress-strain relationship of concrete is derived based on the geometric and material properties of the underlying lattice structure. The derivation of this model is based on the approach used in granular mechanics. Analyses are made using these two approaches to compare with experiments on specimens with double notches loaded in uniaxial tension and shear. These tests are used to evaluate the performance and applicability of these two models. It seems that the Lattice Model is more appropriated for understanding the failure mechanisms of concrete, while when global predictions (like load-CMOD diagrams) are needed, the Micropolar Model performs better. It seems more appropriated to first deal with correct predictions of fracture mechanisms, and then proceed with adjusting the global load-CMOD diagrams. Suggestions about the improvements of these two models will also be discussed.

Keywords: Micropolar Model; Lattice Model; Fracture; Concrete

1. Introduction

Stress-strain relationships used for modeling the mechanical behavior of materials have been traditionally derived follow a phenomenological approach, without explicit considerations of the microstructures of material. We have previously adopted a microstructural mechanics approach to model the development and propagation of fracture in concrete [7], which is assumed to have an underlying microstructure of lattice type. Although the lattice does not directly reflect the microstructure of concrete, it has been demonstrated as an useful model for the description of concrete fracture [14, 15], in particular when a microstructure is projected on a lattice and corresponding properties are assigned to relevant elements in the lattice. The lattice merely serves as a convenient computational backbone.

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In the Microstructural Micropolar Model, the stress-strain relationship of the concrete is derived based on the geometric and material properties of the underlying lattice structure. Since a lattice network of beams can be represented by a lattice network of particles, we adopt the approach used in granular mechanics [2-4, 6, 11, 19].

Another microstructural approach very efficient to simulate fracture processes in concrete is the Delft Lattice Model developed by Schlangen and Van Mier [14]. In this model, concrete is schematized as a network of two-node beams. Normally, three main phases (aggregate, matrix and bond zone) are considered. The heterogeneity of concrete is introduced by mapping different material properties on different lattice beam elements. The fracture process is simulated by removing the element with the highest stress/strength ratio in each loading step from the lattice structure.

In the current paper, we first briefly show the outlines of the two microstructural approaches. Then, numerical simulations of specimens under uniaxial tension tests and shear box tests are presented to evaluate the performance and applicability of these two models using these two different approaches. Finally, the application of these two models will be discussed with suggestions about the improvements of them.

2. Delft Lattice Model

In the “Delft lattice model” concrete is schematized as a network of two noded Bernoulli beams [14, 15, 21]. The nodes of the lattice are arranged according to a regular or random distribution. In the former case they are vertices of equilateral triangles; while in the later case the connectivities between the nodes are created with the Voronoi construction. As concrete is a heterogeneous material, a certain material disorder should be taken in account. And the most straightforward way to include effects from heterogeneity is through direct implementation of disorder. Several methods can be distinguished [15, 16]. The most common way to include the material heterogeneity is to superimpose the mesh (lattice) on top of a computer generated or digital image of a real concrete structure [15, 16]. For concrete often three material phases are distinguished, namely aggregate, matrix and interfacial transition zone (bond). Different stiffness and strength are assigned to the beams that fall in each phase. By removing in each loading step the beam element with the highest stress over strength ratio, fracture is simulated. The basic assumption is that the beam elements have a linear elastic behavior up to failure, and fail in a purely brittle manner. Despite the brittleness of the single elements, global structural softening can be simulated. In the lattice model, some relevant geometrical and material parameters should be determined. A simple fracture criterion is used as described in beams [15, 21], which is based on the effective stress.

\[
\sigma_{eff} = \beta \cdot \left( \frac{N}{A} \pm \alpha \cdot \frac{\left( |M_i|, |M_j| \right)_{\max}}{W} \right) \leq f_{i,k}
\]  

In this equation \(N\) is the normal force acting in the considered beam element, and \(M_i\) and \(M_j\) are the bending moments in the two nodes. \(W\) is the section modulus (\(W=bh^2/6\)), and \(\alpha\) is a factor that regulates the amount of bending that is taken into account. In this paper, \(\alpha = 0.005\) is adopted for all analyses. The parameter \(\beta\) in the fracture law is used to scale the numerical results such as the maximum load to the maximum load in the experiments. The inequality is considered for all the beams in phases \(k\) in the material, and the most critical beam is removed. Essential is to remove only one beam at each loading step to follow stress-redistribution as accurately as possible.
3. Microstructural Micropolar Model

A set of randomly located points can be represented as a lattice network by connecting these points with lines. If the lines are regarded as beams, the lattice network becomes a frame structure which has been demonstrated as an useful model for the description of concrete fracture [15], in particular when a microstructure is projected on a lattice and corresponding properties are assigned to relevant elements in the lattice.

Alternatively the points can be considered as particles which are connected by springs. In general, three different types of spring are used to represent the stiffness between two contact particles. They are stretch spring, shear spring and rotational spring. The lattice network of particles is used extensively in granular mechanics to represent a granular medium [1, 5, 9, 22]. In principle, the representation of a lattice network of beams is equivalent to a lattice network of particles. In our previous paper [7], we described the interaction of two particles in a lattice network and then demonstrated that the behavior of a lattice beam can be represented by a system of two particles connected by springs. It was also shown that the stiffness matrix of two-particle system connected by springs can be identical to that of a lattice beam if certain relationships hold true between the properties of beam and inter-particle springs.

As a lattice network of beams and a lattice network of particles are discrete systems, we can treat the lattice network of beams as an ‘equivalent’ continuum and then derive the stress-strain relationship of the continuum based on the geometric and material properties of the lattice structure. The stress-strain relationship obtained proved to be equivalent with that of Cosserat model [3, 18] in the plane stress condition. Bases on the current model, the internal length of the material can be also derived from its underlying lattice structure which plays an important role in regularization for achieving mesh independency in a finite element method [18].

The damage in this continuum is dictated by the damage of its underlying lattice structure according to a fracture law established below:

\[
\frac{F}{A_b} + \frac{|V|}{A_b} > 1
\]

(2)

where \(F\) is the axial tensile force and \(V\) is the shear force applied to the lattice beam underlying each continuum element, \(f_t\) is tensile strength and \(f_{sh}\) is shear strength of the underlying lattice beams and \(A_b\) is the cross section area of beams.

For the case that considers only tensile failure mode,

\[
\frac{F}{A_b} > 1
\]

(3)

For the case that considers only shear failure mode,

\[
\frac{|V|}{A_b} > 1
\]

(4)

Since the underlying lattice network has three directions of lattice beams, the fracture criterion has to be checked in all three directions. As soon as the beam reached its allowable
tensile strain, it is broken and completely loses its load bearing capacity and then it will not contribute in the stiffness matrix. No softening law is imposed.

For example of the lattice structure shown in figure 1a, after one set of the beams has reached its allowable strength, these beams lost their bearing capacity (see the gray lines in Fig. 1b). Then, just two set beams are active to support the loads. The stiffness matrix obtained considering the damage becomes anisotropic but it still shows a load bearing capacity in the direction in which the beam is broken. The strongest material axis for this damaged material is not perpendicular to the direction of damage. This is a feature that differs from the usual damage models. In the same manner, after the failure of two sets of beams (see Fig. 1c), the three sets of load bearing members are reduced to one. The stress-strain matrix becomes singular which means that the material lost completely its load bearing capacity in the direction perpendicular to the intact (non-broken) beams, even though it can still carry load along the direction of intact beams.

![Figure 1. Original and damaged underlying lattice structures of a representative volume](image)

The crack representation in continuum elements (3-noded triangular element) is represented by drawing a normal crack from the centroid of the element to the side where the underlying lattice beams in the same direction failed.

4. Numerical Simulations

Here we present numerical simulations using the two different microstructural approaches to compare with uniaxial tensile tests and mixed-mode fracture tests on specimens with double off-centered notches.

4.1 Uniaxial tensile test on double notched specimens

In this case, we simulate the experiments of uniaxial tensile tests on double notched specimens carried out at TUDelft [17]. The specimen size is 60×120×10 mm³ with a notch of 10×2 mm². Here, we use the Micropolar Model to simulate the crack evolution and the numerical results obtained will be compared with that of Delft Lattice Model and the experimental data. The finite element meshes are shown in Figure 2 for two different configurations: the vertical off-centered distances chosen between the two notches are 0 and 15 mm, symmetric with respect to the horizontal center axis of the specimen. For this analysis, three meshes (872 elements, 1206 elements, and 1664 elements) are made for the case of zero off-centered distance between the two notches (0 mm). For the three meshes, the lattice structure alignments are assigned randomly to elements. The results obtained from three meshes are compared to evaluate the effects of the mesh sizes and lattice structure alignment. The input material properties are given in Table 1.
Figure 2. (a) Three mesh discretizations for specimen with double notches and (b) mesh for specimen with 15cm off-centered distance between the two notches

<table>
<thead>
<tr>
<th>E (MPa)</th>
<th>ν (Poisson ratio)</th>
<th>f (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21220</td>
<td>0.15</td>
<td>4.16</td>
</tr>
</tbody>
</table>

The crack zones of the four specimens obtained from the finite element analyses are shown in figure 3. The crack patterns as shown in figure 3 and the evolution of strain contour can be illustrated in figure 4. Due to the effect of random lattice alignments, for the double notched specimen with zero off-centered distance, there are three different crack patterns: the up-curved crack, the straight crack and the s-shape crack. All three patterns resemble the observed crack patterns in experiments. The randomly aligned lattice structures reflect indirectly the aggregate arrangements in a concrete, thus produce different crack paths. In figure 3(d), the crack pattern of the specimen with off-centered notches shows a bridge of two cracks developed on both sides. This is also typical for observed crack pattern in the experiment (see Shi et. al. [17]).

Figure 3. Comparison of crack zones
Figure 4. Crack patterns in the four cases

The same test had been simulated by Shi et. al. [17] using the Delft Lattice Model with specimen of same dimensions. The specimen was modeled with regular triangular lattices in the area where the cracks are expected, while the rest of the area is modeled with 4-noded quadrilateral continuum elements. The length of a lattice element is set to 1mm. The aggregate sizes vary between 2mm to 8mm. The volume ratio of aggregate $P_k = 0.7$. The material and geometrical parameters, i.e. tensile strength $f_t$, Young’s modulus $E$ and Poisson ratio $\nu$, are determined according to refs. [15, 21]: $f_{t,a} = 10\text{MPa}$, $f_{t,m} = 5\text{MPa}$, $f_{t,b} = 1.7\text{MPa}$, where $a$, $m$ and $b$ denote aggregate, matrix and bond respectively. The elastic properties $E_a = 66429\text{MPa}$ and $E_m = E_b = 22143\text{MPa}$ and $\nu = 0.16$.

The figure 5 showed the calculated crack patterns for two types of notch off-set (0mm and 15mm). With increasing load, discrete microcracks start in the center area by debonding of the aggregates and matrix. At some stage, cracks form from two notches and propagate to the opposite side. It seems that these two cracks attract each other when the tips are further apart, whereas they repel each other when the tips are closer, which is in agreement with experiments. The figure 8 showed the load-deformation diagram obtained from lattice simulation and the experiments. The figure 6 showed the crack patterns of three different aggregate structures for the specimen with 15mm notch off-set. It can be observed that the crack pattern is aggregate dependent and displayed arbitrary characteristics for different aggregate structures.

Figure 5. Crack patterns for 2 types of notch off-set, using Lattice Model (Shi et. al. [17])

Figure 6. Crack patterns of 15mm notch off-set specimen with 3 different types of aggregate structures, using Lattice Model (Shi et. al. [17])
The measured vertical displacement of two points 30 mm apart across the notches at the left and right edge of the specimen is plotted against load in figure 7a for specimen with zero off-centered notches and in figure 7b for 15 mm off-centered notches. The experimental results are also compared with the computed results from finite element analysis. According to figure 7a, the finite element results of Micropolar Model show that different mesh discretizations have little influence upon the structural behavior. The strength ranges approximately 10% for the three meshes. For the case of 15mm off-centered notches, the strength is about 15% lower than observed in experiment. The predicted range is within the range of experimental results. It can be observed that the both models give load-displacement curves more brittle than the experimental one, especially the lattice results.

![Graph](image)

**Figure 7.** Load-displacement curves for specimen with: a) zero off-centered notches of different mesh sizes and b) 15mm off-centered notches

![Graph](image)

**Figure 8.** Load-displacement curves for specimen with 15mm off-centered notches using Delft Lattice Model (see Shi et. al.,[17])

### 4.2 Mixed-mode fracture test

Figure 9 schematically shows the configuration of the mixed-mode concrete fracture device used for the experiments performed by Nooru-Mohamed [13]. The double-edge-notched (DEN) specimen was placed in a special loading frame to allow for the application of various loading paths using either a force or a deformation control method. The behavior was measured for the specimen under a combined shear stress and tensile stress.
Attention is focused on the largest specimen used in the experiments, for which the box height \( L = 200\text{mm} \), the thickness \( t = 50\text{mm} \) and the size of the two symmetrical notches is \( 25 \times 5 \text{ mm} \). The specimen is supported at the bottom and along the right hand side below the notch. The shear force \( P_s \) was first applied through the frame above the notch along the left hand side of the specimen. Then the tensile force \( P \) was applied at the top (the frame was glued to the specimen). The relative tensile displacement \( \delta \) was measured between the points A and A’ on the left side as well as the points B and B’ on the right side of the shear box.

Numerical simulations using the Micropolar Model were made for two experiments conducted by Nooru-Mohamed [13] corresponding to two values of the shear force: \( P_s = -5\text{kN} \) and \( P_s = -10\text{kN} \). In the two simulations the shear force \( P_s \) is applied using a force control method and then kept constant while the normal tensile loading is imposed using a deformation control method, which corresponds the loading path 4 from the experiments by Nooru-Mohamed [13]. The specimens used in these experiments were made from normal weight concrete with an averaged aggregate size of 2mm. In this shear box analysis, the fracture criterion used is Eq. (2), which considers both tensile and shear failure modes. The material parameters used in numerical simulation are as follow: Young’s modulus \( E = 15000 \text{ N/mm}^2 \), Poisson’s ratio \( \nu = 0.2 \), tensile strength \( f_t = 4.65 \text{ N/mm}^2 \), and shear strength \( f_{sh} = 4.65 \text{ N/mm}^2 \). It is noted that in the following analysis, we select \( f_{sh} = f_t \).

The finite element mesh for the double-notched specimen is shown in Figure 9. For the nodal points on the top and the bottom, the displacements are fixed in \( y \) direction and the rotations are fixed. Corresponding to \( P_s \), forces are applied on the nodal points at the lower-right side and at the upper-left side.

![Figure 9. Double-edge-notched specimen: (a) shear box configuration and applied shear load; (b) finite element mesh used for numerical simulation](image-url)
According to the experimental results obtained by Nooru-Mohamed [13], the peak axial tensile load was 15kN for the case of low shear load (i.e., $P_s = -5$kN). A 30% decrease in $P$ was observed when $P_s$ was increased from -5kN to -10kN. Nooru-Mohamed justified this result by the fact that the presence of a lateral shear load significantly affected the axial tensile capacity of the specimen. It is noted that the fracture criterion used in this analysis (i.e., Eq. 2), the presence of a lateral shear load does have effects on the axial tensile capacity of the specimen. Figure 10(a) and (b) shows the crack patterns observed from the experiments by Nooru-Mohamed [13] for the two cases of shear loads. The specimen with $P_s = -5$kN shows a crack pattern similar to the direction tensile test. The cracks initiated at the notched area and then developed in horizontal direction. For the specimen with $P_s = -10$kN, the crack pattern exhibits in a different form. The cracks initiated at the notched area but developed along an inclined direction. The cracks propagate and then are curved back to the horizontal direction.

![Crack patterns from experiments](image)

(a)

(b)

Figure 10. Crack patterns from experiments: (a) $P_s = -5$kN and in (b) $P_s = -10$kN (Nooru-Mohamed [13])

The analyses using micropolar model fits quite well the experimental results. Figure 11a showed the comparison of load-displacement curves between experimental and numerical results, using internal length $l_{int} =1$mm. According to the numerical simulation, the peak axial tensile load was 15kN for the case of low shear load (i.e., $P_s = -5$kN). And approximately a 20% decrease in $P$ was observed when $P_s$ was increased from -5kN to -10 kN. The rotation degree of freedom allows the effect of fracture criterion used in this analysis (i.e., Eq. 2) to reflect on the magnitude of peak load. The values of shear strength $f_{sh}$ in the fracture criterion indicates the degree of shear effect on the tensile capacity. Therefore it has a significant effect on the differences of the two peak loads.

The same test had been carried out by Schlangen [15], using the Delft Lattice Model. The specimen was modeled with random triangular lattices which is another way to represent material heterogeneity. Figure 11b showed the comparison of load-displacement curves between experimental and numerical results obtained by Schlangen [15]. It appears that the predicted load-displacement responses do not fit so well the experimental data.
Figure 11. Comparison of load-displacement curves between experimental data and numerical simulation using: a) Micropolar Model \((l_{\text{int}} = 1\text{mm})\) and b) Delft Lattice Model

Figure 12 shows the crack patterns obtained by numerical simulation, using Micropolar Model. From Figures 12, it is observed that the shear load has significant influence upon the crack pattern. The crack patterns in the simulation are in good agreement with that observed from experiments as shown in Figure 10. This demonstrated that the micropolar model has the advantage over the non-polar model [7] in a more realistic predicted behavior. The added degree of rotation and the ability of transmitting couple stress in the medium are the mechanisms that are realistic in the material, especially in a specimen under both shear and tensile stresses. Figure 13 shows the crack patterns obtained by numerical simulation, using the Delft Lattice Model. The crack patterns in the simulation are also in good agreement with that observed from experiments as shown in Figure 10.

Figure 12. Comparison of crack patterns from experiments (fig. 10a and 10b) and numerical simulations: (a) \(P_s = -5\text{kN}\) and (b) \(P_s = -10\text{kN}\) \((l_{\text{int}} = 1\text{mm})\)
5. Discussion

5.1 Delft Lattice Model

It is shown that the Delft Lattice Model is capable of simulating the fracture process in concrete correctly, especially when (local) mode I failure prevails. An example of such a case is the uniaxial tensile test. The simulated results of this test are in good agreement with the outcome of the experiments by Shi et. al. [17]. Firstly, microcrack occurs in the material which starts around aggregates. Beyond peak load the crack starts to localize. At the end of the steep drop in the descending branch the crack has traversed the cross-section completely. The long tail can be explained by crack face bridging.

One of the deficiency of this model is that the numerical load-displacement curves are too brittle in comparison with the experimental curves. Schlangen [15] justified this problem by the fact that the small particles were not included in the computer generated grain structure. For example, in the uniaxial tensile test, only particles of 2mm and larger were included. Then, the amount of crack face bridging may be not sufficient, and gives a too brittle load-displacement response. For including small particles in the generated grain structure, the beam length has to be reduced, which increase greatly the computational effort. Next to the small particle effect, also the influence of two dimensional modelling is another factor which causes the global brittleness.

Another deficiency is the enormous computational effort required to perform finite element analysis, because the individual beam length should be smaller than (at least) 1/3 of the minimum aggregate of the specimen to get a reasonable load-displacement curve in comparison with the experimental data. This limits the application of lattice model in large-scale structures. Recently, efforts have been made to apply the Delft Lattice Model in the large-scale structures. One of the solutions is to employ statistical distributions to introduce the material heterogeneity on coarser meshes without discretized aggregate structures. The strength of the beam elements is the most obvious choice as parameter in a statistical distribution. Several examples of the use of statistical distributions can be found in [8, 10, 15]. Recently, a systematic survey of the effect of a certain statistical distribution on fracture processes in concrete has been carried out by Tai K. et. al. [20]. In this research Gaussian and Weibull distributions are employed to evaluate the effect of different scales of heterogeneity. It has been proved that the Weibull distribution can simulate better the force-displacement curves obtained from experiments. But, the material heterogeneity implemented in this method can not represent the real material disorder. Consequently, we can not simulate the failure mechanisms properly. Another way to make possible the application of lattice model
in large-scale structures is the employment of parallel computer technique which is being implemented in TUDelft.

An improvement should be made in cases where compression plays an important role or dominates. An example of such a case is the mixed-mode test. If the two curved cracks are arrested in the compression zone, the splitting crack in the middle starts to grow. When the crack in the middle starts, the two curved cracks close again. Yet, closure of the cracks would imply transfer of compressive load again. This is not possible, because of the removal of the beam elements. To overcome this, a check should be implemented in which the distance between two nodes which were connected by a beam is calculated. If in a certain loading step this distance becomes shorter than the original beam length, the element must be placed back, and the loading step repeated.

5.2 Micropolar Model

For the microstructural mechanics approach, it is interesting to note that for a conventional micropolar model without the consideration of underlying structure (e.g., Muhlhaus et al., [12]), the results are insensitive to mesh sizes for a shear test. But it is sensitive to mesh size for uniaxial tensile test. The present micropolar model shows that it has low sensitivity to the mesh size in all modes including the condition of uniaxial tensile test. Therefore, the microstructural consideration can be regarded as a useful regularized technique.

Although the finite element results of this model are not influenced by the mesh size, they are influenced by the alignment of the underlying lattice structure. The influence is not much in the elastic range. However, the alignment causes anisotropy in strength thus has significant influence on the peak strength and after peak behavior. It may be viewed that the alignment of a lattice structure reflects the arrangement of aggregates in concrete. For simulating a real material, we assume that the lattice alignment in each element is random. The range of computed results due to the random microstructure should be comparable to the range of measured experimental results.

According to the analysis for the mixed mode experiments, the micropolar model is capable of capturing the features of the measured crack patterns and the influence of shear load on the tensile peak loads in the load-displacement curves, while a conventional non-polar model [7] can not capture the same phenomena. Because, especially in the loading cases with dominant shear stress, the behavior of the specimen is greatly influenced by the internal length, which is ignored in the non-polar model.

In this model, the numerical load-displacement response is more brittle than the experimental, which can be improved by the implementation of a softening law in the present model.

5.3 Final remark

It has been shown that Lattice Model can simulate better the fracture process in concrete correctly for (local) mode I failure. To make the Lattice Model general accessible for modelling concrete fracture, aggregate interlock needs to be considered. This can be achieved by allowing the removed beams to transfer compression and (limited) shear forces, while the tensile capacity of beams is neglected. Such a modification implies a nonlinear constitutive relation for the removed beams, and will therefore increase significantly the calculation time for finite element analysis.
For the Micropolar Model, the predicted load-displacement response is more brittle than the experimental data. To get a better match, a softening law for the removed beams could be implemented. To capture the failure mechanism as the Lattice Model does, a computer generated grain structure could be projected upon the underlying lattice structure and then the correspondent properties could be assigned to the beams.

It seems that the Lattice Model is more appropriated for understanding the failure mechanisms of concrete, while when global predictions (like load-CMOD diagrams) are needed, the Micropolar Model performs better. It seems more appropriated to first deal with correct predictions of fracture mechanisms, and then proceed with adjusting the global load-CMOD diagrams.

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References


